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Title: WIRELESS SIX-DEGREE-OF-FREEDOM LOCATOR

FIELD AND BACKGROUND OF THE INVENTION

The present invention relates to a method for monitoring the position and orientation of a moving object, of the type in which the moving object transmits electromagnetic signals, representative of the position and orientation thereof, to a fixed receiver. More particularly, the present invention relates to an open loop method in which either the transmitter or the receiver may be spatially extended and in which both the position and the orientation of the moving object are computed noniteratively.

It is known to track the position and orientation of a moving object with respect to a fixed frame of reference, by equipping the moving object with a transmitting apparatus that transmits electromagnetic radiation, placing a receiving apparatus in a known and fixed position in the fixed frame of reference, and inferring the continuously changing position and orientation of the object from signals transmitted by the transmitting apparatus and received by the receiving apparatus. Typically, the transmitting apparatus includes three orthogonal magnetic dipole transmitters; the receiving apparatus includes three orthogonal magnetic dipole receivers; and the object is close enough to the receiving apparatus, and the frequencies of the signals are sufficiently low, that the signals are near field signals. Also typically, the system used is a closed loop system: the receiving apparatus is hardwired to, and explicitly synchronized with, the transmitting apparatus. Representative prior art patents in this field include US 4,287,809 and US 4,394,831, to Egli et al.; US 4,737,794, to Jones; US 4,742,356, to Kuipers; US 4,849,692, to Blood; and US 5,347,289, to Elhardt. Several of the prior art patents, notably Jones,

present non-iterative algorithms for computing the position and orientation of magnetic dipole transmitters with respect to magnetic dipole receivers.

Of particular note are US 4,054,881, to Raab, and US 5,600,330, to Blood. Raab purports to teach an open loop system. Raab's system is "open loop" only in the sense that there is no communication from the receiving apparatus to the transmitting apparatus; but it still is necessary to synchronize the transmitting apparatus and the receiving apparatus explicitly. Raab provides several methods for synchronizing the receiving apparatus with the transmitting apparatus, for example a phase locked loop in the case of frequency domain multiplexing, and code timing signals, in the case of spread spectrum multiplexing. In all cases, however, Raab's system requires that the receiver generate a reference signal that is mixed with the received signal, both for the purpose of synchronization and for the purpose of resolving sign ambiguities in all three independent coordinates of the space in which the object moves. In Blood's system, the transmitters are fixed in the fixed reference frame, and the receivers are attached to the moving object; but by reciprocity, this is equivalent to the situation in which the receivers are fixed and the transmitters move. Blood's transmitters are spatially extended, and so cannot be treated as point sources. Blood also presents an algorithm which allows the orientation, but not the position, of the receivers relative to the transmitters to be calculated non-iteratively.

It thus is apparent that there is further room for simplification of the art of tracking a moving object using near field electromagnetic signals. The explicit synchronization required by Raab demands additional hardware and/or signal processing that would not be necessary if explicit synchronization were not required. Blood's ~~non-iterative~~ iterative calculation of position adds complexity and processing time, to systems with spatially extended transmitters or receivers, that are absent from systems

with point sources and point receivers. It would be highly advantageous to have a noniterative method of inferring both the position and the orientation of a transmitting apparatus relative to a spatially extended receiving antenna without explicit synchronization of the transmitters and the receivers.

SUMMARY OF THE INVENTION

According to the present invention there is provided a method for determining the position and orientation of an object with respect to a reference frame, including the steps of: (a) providing the object with three independent transmitters of electromagnetic radiation; (b) providing three independent receivers of the electromagnetic radiation, each of the receivers having a fixed position in the reference frame; (c) transmitting the electromagnetic radiation, using the transmitters, a first of the transmitters transmitting the electromagnetic radiation including at least a first frequency, a second of the transmitters transmitting the electromagnetic radiation including at least a second frequency different from the first frequency, and a third of the transmitters transmitting the electromagnetic radiation including at least a third frequency different from the first frequency; (d) receiving signals corresponding to the electromagnetic radiation, at all three of the receivers, at a plurality of times, each of the signals including components of at least one of the three frequencies; (e) for each of the receivers, forming a first function of the components including the components of the signal received by the each receiver from the first transmitter at the first frequency, a function of the components including the components of the signal received by the each receiver from the second transmitter at the second frequency, and a function of the components including the components of the signal received by the each transmitter from the third transmitter at the third frequency, the functions being

independent of a time delay between the transmitters and the receivers; and (f) inferring the position and the orientation of the object from the functions.

According to the present invention there is provided a method for determining the position and orientation of an object with respect to a reference frame, including the steps of: (a) providing the object with three independent transmitters of electromagnetic radiation; (b) providing three independent receivers of the electromagnetic radiation, each of the receivers having a fixed position in the reference frame, at least one of the receivers being spatially extended; (c) transmitting the electromagnetic radiation, using the transmitters, a first of the transmitters transmitting the electromagnetic radiation including at least a first frequency, a second of the transmitters transmitting the electromagnetic radiation including at least a second frequency different from the first frequency, and a third of the transmitters transmitting the electromagnetic radiation including at least a third frequency different from the first frequency; (d) receiving signals corresponding to the electromagnetic radiation, at all three of the receivers, at a plurality of times; and (e) inferring the position and the orientation of the object noniteratively from the signals.

Figure 1 shows schematically the hardware of the present invention. A moving object **10** is provided with three independent magnetic dipole transmitter coils **12**, **14** and **16** that are powered by transmission circuitry **18**. Fixed within the reference frame with respect to which object **10** moves are three independent, spatially extended receiver antennas **20**, **22** and **24**, electrically coupled to reception circuitry **26**. As defined herein, "independent" means that the time varying magnetic fields created by one of coils **12**, **14** or **16** cannot be expressed as a linear combination of the time varying magnetic fields created by the other two coils, and that the time varying signals received by one of antennas **20**, **22** or **24** cannot be expressed as a linear

combination of the signals received by the other two antennas. Preferably, coils **12**, **14** and **16** are mutually orthogonal, as shown in Figure 1. In the most preferred embodiments of the present invention, antennas **20**, **22** and **24** are coplanar, as shown in Figure 1.

Although only one tracked object **10** is illustrated in Figure 1, it will be readily appreciated from the description below that the present invention is easily adapted to the simultaneous tracking of several objects **10**.

Transmission circuitry **18** and reception circuitry **26** need not be explicitly synchronized, as long as the clocks of transmission circuitry **18** and reception circuitry **26** do not drift with respect to each other during one measurement of the position and orientation of object **10** with respect to antennas **20**, **22** and **24**. This requirement is easily achieved using clocks based on modern crystal oscillators. Two algorithms are presented below whereby signals received by antennas **20**, **22** and **24** at a plurality of reception times are transformed into a 3 x 3 matrix $\boldsymbol{M}$ that is independent of any unknown time shift Δ between the clock of transmission circuitry **18** and reception circuitry **26**. One of these algorithms requires synchronization of transmission circuitry **18** and reception circuitry **26** at the beginning of a sampling cycle in order to resolve a phase ambiguity in the matrix $\boldsymbol{M}$; the other algorithm needs no such synchronization, and resolves the phase ambiguity using the phases of the received signals. A third algorithm is presented below whereby a rotationally invariant 3 x 3 position matrix $\boldsymbol{W}$ and a 3 x 3 rotation matrix $\boldsymbol{T}$ are inferred noniteratively from the matrix $\boldsymbol{M}$. The Euler angles that represent the orientation of object **10** relative to the fixed frame of reference are calculated noniteratively from the elements of $\boldsymbol{T}$, and the Cartesian coordinates of object **10** relative to the fixed frame of reference are

calculated from the elements of ***W***. A preliminary calibration of the system, either by explicitly measuring the signals received by antennas **20**, **22** and **24** at a succession of positions and orientations of object **10**, or by theoretically predicting these signals at the successive positions and orientations of object **10**, is used to determine polynomial coefficients that are used in the noniterative calculation of the Euler angles and the Cartesian coordinates.

BRIEF DESCRIPTION OF THE DRAWINGS

The invention is herein described, by way of example only, with reference to the accompanying drawings, wherein:

FIG. 1 is a schematic overall depiction, partly in perspective, of the hardware of the present invention;

FIG. 2 is a schematic diagram of a preferred embodiment of the transmission circuitry of the present invention;

FIG. 3 is a schematic diagram of a simpler preferred embodiment of the transmission circuitry of the present invention;

FIG. 4 is a schematic diagram of a preferred embodiment of the reception circuitry of the present invention;

FIG. 5 is a schematic diagram of a simpler preferred embodiment of the reception circuitry of the present invention.

FIG. 6 shows the three component antennas of a set of coextensive linearly independent receiving antennas.

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DESCRIPTION OF THE PREFERRED EMBODIMENTS

The present invention is of a method for determining the position and orientation of a target with respect to a reference coordinate frame, by transmitting electromagnetic signals from the target to a receiver that is fixed in the reference coordinate frame, with simpler synchronization of the transmitter and the receiver than in the prior art, or even without explicit synchronization of the transmitter and the receiver, in a system in which the transmitter and receiver are not hardwired. The scope of the present invention also includes a noniterative determination of both position and orientation with respect to spatially extended receiver antennas.

The principles and operation of target tracking according to the present invention may be better understood with reference to the drawings and the accompanying description.

Most generally, the signals transmitted by coils **12**, **14** and **16** must be temporally independent, in the sense that the signal supplied to any one of coils **12**, **14** and **16** by circuitry **18** is not a linear combination of the signals supplied to the other two coils by circuitry **18**. This is achieved most simply and most preferably by transmitting from each coil at a different frequency. For definiteness, the angular frequency of the transmissions from coil **12** is designated herein as ω_1, the angular frequency of the transmissions from coil **14** is designated herein as ω_2, and the angular frequency of the transmissions from coil **16** is designated herein as ω_3.

The transmitted signals induce received signals in antennas **20**, **22** and **24**. Reception circuitry **26** is operative to digitize the received signals at a sequence of times t_m which are preferably but not necessarily equally spaced. It should be noted that this spacing need not be synchronous with the transmission frequencies.

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Conceptually, reception circuitry **26** consists of three receivers, each coupled to a different antenna, and computational means for inferring the position and orientation of target **10** from the signals received by the three receivers. The received signals may be organized in a matrix s of three rows, one row for each receiver, and as many columns as there are times t_m, one column for each time. Each element of s can be written as:

$$s_{im} = c_{i1}\cos\omega_1 t_m + c_{i2}\sin\omega_1 t_m + c_{i3}\cos\omega_2 t_m + c_{i4}\sin\omega_2 t_m + c_{i5}\cos\omega_3 t_m + c_{i6}\sin\omega_3 t_m \qquad (1)$$

c_{i1}, c_{i3} and c_{i5} are the in-phase components of the signals received by receiver i from coils **12**, **14** and **16**, respectively. c_{i2}, c_{i4} and c_{i6} are the quadrature components of the signals received by receiver i from coils **12**, **14** and **16**, respectively. Note that components c_{i1} and c_{i2} refer to frequency ω_1, components c_{i3} and c_{i4} refer to frequency ω_2, and components c_{i5} and c_{i6} refer to frequency ω_3. The components c_{ij} can themselves be arranged in a matrix $\boldsymbol{c}$ of three rows and six columns. The matrices **s** and **c** are related by a matrix **A** of six rows and as many columns as there are in matrix **s**:

$$\boldsymbol{s} = \boldsymbol{cA} \qquad (2)$$

Because the transmission frequencies and the reception times are known, matrix $\boldsymbol{A}$ is known. Equation (2) is solved by right-multiplying both sides by a right inverse of matrix $\boldsymbol{A}$: a matrix, denoted as $\boldsymbol{A}^{-1}$, such that $\boldsymbol{AA}^{-1} = \boldsymbol{I}$, where $\boldsymbol{I}$ is the 6x6 identity matrix. Right inverse matrix $\boldsymbol{A}^{-1}$ is not unique. A particular right inverse matrix $\boldsymbol{A}^{-1}$ may be selected by criteria that are well known in the art. For example, $\boldsymbol{A}^{-1}$ may be the right inverse of $\boldsymbol{A}$ of smallest L^2 norm. Alternatively, matrix **c** is determined as the generalized inverse of equation (2):

$$\boldsymbol{c} = \boldsymbol{s}\boldsymbol{A}^T(\boldsymbol{A}\boldsymbol{A}^T)^{-1} \qquad (3)$$

where the superscript "T" means "transpose". The generalized inverse has the advantage of being an implicit least squares solution of equation (2).

Whether the right inverse A^{-1} or the generalized inverse $A^T(AA^T)^{-1}$ is used to solve equation (2), the right-multiplication of matrix s constitutes a digital filtering operation that returns the amplitudes and phases of the received frequency components in the form of the elements of matrix c. In general, the elements of matrix c must be corrected for amplitude and phase distortions introduced, for example, by reception circuitry **26**. This can be done easily given the transfer functions of reception circuitry **26**. In the following discussion, it is assumed that such corrections have been made.

The present invention includes two preferred algorithms for forming the matrix M from the matrix c. In the first algorithm, each column of M is formed from components corresponding to the same frequency. Let receiver i' be the receiver with the largest signal magnitude at frequency ω_j. In other words, at one particular value of j,

$$(c^2_{i,2j-1} + c^2_{i,2j})^{1/2} \tag{4}$$

is largest for $i=i'$. Then

$$M_{i'j} = (c^2_{i',2j-1} + c^2_{i',2j})^{1/2} \tag{5}$$

and the other two elements ($i \neq i'$)of the j-th column of M are

$$M_{ij} = (c_{i,2j-1}c_{i',2j-1} + c_{i,2j}c_{i',2j})/M_{i'j} \tag{6}$$

The reference to the receiver with the largest signal magnitude tends to suppress noise. The fact that the two matrix elements of the j-th column of M for $i \neq i'$ are projections of their signals onto the signal of largest magnitude tends to suppress eddy current noise, which tends to be 90° out of phase with the signal.

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In the second algorithm, frequencies ω_2 and ω_3 are chosen to be even multiples of frequency ω_1, and all matrix elements are referred to the strongest signal at frequency ω_1. The first column of M is as in the first algorithm. Let $\omega_2 = \xi\omega_1$ and $\omega_3 = \zeta\omega_1$, where ξ and ζ are even integers. Note that the matrix

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$$R = \begin{pmatrix} c_{i',1} & c_{i',2} \\ -c_{i',2} & c_{i',1} \end{pmatrix} / \left(c_{i',1}^2 + c_{i',2}^2\right)^{1/2} \tag{7}$$

is a rotation matrix. The elements of the second column of M are the first elements of the column matrices obtained by the operation

T0111

$$R^{\xi}\begin{pmatrix} c_{i3} \\ c_{i4} \end{pmatrix} \tag{8}$$

for i=1,2,3. The elements of the third column of M are the first elements of the column matrices obtained by the operation

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$$R^{\zeta}\begin{pmatrix} c_{i5} \\ c_{i6} \end{pmatrix} \tag{9}$$

for I=1,2,3.

It is straightforward to show that as long as the transmitter clock and the receiver clock do not drift relative to each other, these expressions for the elements of matrix M are independent of any time shift Δ between the transmitters and the receivers: substituting t_m+Δ for t_m in equation (1) does not change the value of the matrix elements of M. If not for the sign ambiguity of the square roots, there would be no need to synchronize the receivers with the transmitters. In fact, under the first algorithm for forming M, it is necessary to synchronize the receivers with the transmitters, as described below, to resolve the sign ambiguity of each $M_{i'j}$. Under the second algorithm for forming M, the remaining ambiguity is resolved as described below.

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Note that under the second algorithm, there is only one sign ambiguity. This can be explained as follows: Assign the signal of frequency ω_1 an arbitrary phase ϕ. Then, if all three transmissions are synchronized, under the near field approximation, the phase of the signal of frequency $\omega_2=\xi\omega_1$ is $\xi\phi$ and the phase of the signal of frequency $\omega_3=\zeta\omega_1$ is $\zeta\phi$. The sign ambiguity of $(c^2_{i',1}+c^2_{i',2})^{1/2}$ is equivalent to an ambiguity of π radians in ϕ. But then the phase of the signal of frequency ω_2 is unambiguously $\xi\phi+\xi\pi = \xi\phi$ modulo 2π and the phase of the signal of frequency ω_3 is unambiguously $\zeta\phi + \zeta\pi = \zeta\phi$ modulo 2π, because ξ and ζ are even integers.

Alternatively, frequencies ω_2 and ω_3 may be the same even multiple ξ of ω_1. Coil **12** transmits a signal proportional to $\sin\omega_1 t$. Coil **14** transmits a signal proportional to $\sin\xi\omega_1 t$. Coil **16** transmits a signal proportional to $\cos\xi\omega_1 t$. Matrix $\boldsymbol{A}$ has only four rows, two for frequency ω_1 and two for frequency $\xi\omega_1$. The second column of $\boldsymbol{M}$ is formed as above. The elements of the third column of $\boldsymbol{M}$ are the *second* elements of the column matrices obtained by the operation $\left[\boldsymbol{R}^{\xi}\begin{pmatrix} c_{i3} \\ c_{i4} \end{pmatrix}\right]$, for i=1,2,3.

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Let $\boldsymbol{T}$ be the orthonormal matrix that defines the rotation of object **10** relative to the reference frame of antennas **20, 22** and **24**. Write $\boldsymbol{M}$ in the following form:

$$\boldsymbol{M} = \boldsymbol{E}\boldsymbol{T}_0\boldsymbol{T} \tag{10}$$

where $\boldsymbol{T}_0$ is an orthogonal matrix and $\boldsymbol{E}$ is in general a nonorthogonal matrix. In general, $\boldsymbol{T}_0$ and $\boldsymbol{E}$ are functions of the position of object **10** relative to the reference frame of antennas **20, 22** and **24**. Let

$$\boldsymbol{W}^2 = \boldsymbol{M}\boldsymbol{M}^T = \boldsymbol{E}\boldsymbol{T}_0\boldsymbol{T}\boldsymbol{T}^T\boldsymbol{T}_0^T\boldsymbol{E}^T = \boldsymbol{E}\boldsymbol{E}^T \tag{11}$$

$\boldsymbol{W}^2$ is real and symmetric, and so can be written as $\boldsymbol{W}^2 = \boldsymbol{P}\boldsymbol{d}^2\boldsymbol{P}^T = (\boldsymbol{P}\boldsymbol{d}\boldsymbol{P}^T)^2$, where $\boldsymbol{d}^2$ is a diagonal matrix whose diagonal elements are the (real and positive) eigenvalues of $\boldsymbol{W}^2$ and where $\boldsymbol{P}$ is a matrix whose columns are the corresponding eigenvectors of $\boldsymbol{W}^2$. Then $\boldsymbol{W} = \boldsymbol{P}\boldsymbol{d}\boldsymbol{P}^T = \boldsymbol{E}$ also is symmetric. Substituting in equation (10) gives:

$$\boldsymbol{M} = \boldsymbol{P}\boldsymbol{d}\boldsymbol{P}^T\boldsymbol{T}_0\boldsymbol{T} \qquad (12)$$

so that

$$\boldsymbol{T} = \boldsymbol{T}_0{}^T\boldsymbol{P}\boldsymbol{d}^{-1}\boldsymbol{P}^T\boldsymbol{M} \qquad (13)$$

If $\boldsymbol{T}_0$ is known, then $\boldsymbol{T}$, and hence the orientation of object **10** with respect to the reference frame of antennas **20**, **22** and **24**, can be computed using equation (13).

The orthogonal rotation matrix $\boldsymbol{T}$ is used to resolve the above-described residual sign ambiguity in the second algorithm for forming $\boldsymbol{M}$. Specifically, the first column of $\boldsymbol{T}$ should be the cross product of the second and third columns of $\boldsymbol{T}$. If, after following the above procedure for forming $\boldsymbol{T}$, the first column thereof comes out as the negative of the cross product of the second and third columns, then the sign of $(c^{2}_{i',1}+c_{2i',2})^{1/2}$ must be reversed.

For any particular configuration of antennas **20**, **22** and **24**, $\boldsymbol{T}_0$ may be determined by either of two different calibration procedures.

In the experimental calibration procedure, object **10** is oriented so that $\boldsymbol{T}$ is a unit matrix, object **10** is moved to a succession of positions relative to antennas **20**, **22** and **24**, and $\boldsymbol{M}$ is measured at each position. The equation

$$\boldsymbol{T}_0 = \boldsymbol{P}\boldsymbol{d}^{-1}\boldsymbol{P}^T\boldsymbol{M} \qquad (14)$$

gives $\boldsymbol{T}_0$ at each of those calibration positions.

There are two variants of the theoretical calibration procedure. In the first variant, coils **12**, **14** and **16** are modeled as point sources, including as many terms in their multipole expansions as are necessary for accuracy, and their transmitted

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magnetic fields in the planes of antennas **20**, **22** and **24** are calculated at a succession of positions relative thereto, also with object **10** oriented so that $\boldsymbol{T}$ is a unit matrix. The EMF induced in antennas **20**, **22** and **24** by these time-varying magnetic fields is calculated using Faraday's law. The transfer function of reception circuitry **26** then is used to compute $\boldsymbol{M}$ at each calibration position, and equation (14) gives $\boldsymbol{T}_0$ at each calibration position. The second variant exploits the principle of reciprocity and treats antennas **20**, **22** and **24** as transmitters and coils **12**, **14** and **16** as point receivers. The magnetic field generated by each antenna at the three frequencies ω_1, ω_2 and ω_3 is modeled using the Biot-Savart law. Note that each frequency corresponds to a different coil. The signal received at each coil is proportional to the projection of the magnetic field on the axis of the coil when object **10** is oriented so that $\boldsymbol{T}$ is a unit matrix. This gives the corresponding column of $\boldsymbol{M}$ up to a multiplicative constant and up to a correction based on the transfer function of reception circuitry **26**.

To interpolate $\boldsymbol{T}_0$ at other positions, a functional expression for $\boldsymbol{T}_0$ is fitted to the measured values of $\boldsymbol{T}_0$. Preferably, this functional expression is a polynomial. In the special case of the "coextensive" preferred embodiment of spatially extended antennas **20**, **22** and **24** described below, it has been found most preferable to express the Euler angles α, β and γ that define $\boldsymbol{T}_0$ as the following 36-term polynomials. The arguments of these polynomials are not direct functions of Cartesian coordinates x, y and z, but are combinations of certain elements of matrix $\boldsymbol{W}^2$ that resemble x, y and z, specifically, $a = W^2_{13}/(W^2_{11}+W^2_{33})$, which resembles x; $b = W^2_{23}/(W^2_{22}+W^2_{33})$, which resembles y, and $c = 1/W^2_{33}$, which resembles z. Using a direct product notation, the 36-term polynomials can be expressed as:

$$\alpha = (a, a^3, a^5)(b, b^3, b^5)(1, c, c^2, c^3)\boldsymbol{AZcoe} \qquad (15)$$

$$\beta = (a, a^3, a^5)(1, b^2, b^4, b^6)(1, c, c^2)\boldsymbol{ELcoe} \quad (16)$$

$$\gamma = (1, a^2, a^4, a^6)(b, b^3, b^5)(1, c, c^2)\boldsymbol{RLcoe} \quad (17)$$

where ***AZcoe***, ***ELcoe*** and ***RLcoe*** are 36-component vectors of the azimuth coefficients, elevation coefficients and roll coefficients that are fitted to the measured or calculated values of the Euler angles. Note that to fit these 36-component vectors, the calibration procedure must be carried out at at least 36 calibration positions. At each calibration position, $\boldsymbol{W}^2$ is computed from $\boldsymbol{M}$ using equation (11), and the position-like variables a, b and c are computed from $\boldsymbol{W}^2$ as above.

Similarly, the Cartesian coordinates x, y and z of target **10** relative to the reference frame of antennas **20**, **22** and **24** may be expressed as polynomials. In the special case of the "coextensive" preferred embodiment of spatially extended antennas **20**, **22** and **24** described below, it has been found most preferable to express x, y and z as the following 36-term polynomials:

$$x = (a, a^3, a^5)(1, b, b^4)(1, c, c^2, c^3)\boldsymbol{Xcoe} \quad (18)$$

$$y = (1, a^2, a^4)(b, b^3, b^5)(1, c, c^2, c^3)\boldsymbol{Ycoe} \quad (19)$$

$$z = (1, a^2, a^4)(1, b^2, b^4)(1, d, d^2, d^3)\boldsymbol{Zcoe} \quad (20)$$

where ***Xcoe***, ***Ycoe*** and ***Zcoe*** are 36-component vectors of the x-coefficients, the y-coefficients, and the z-coefficients, respectively; and $d = \log(c)$. As in the case of the Euler angles, these position coordinate coefficients are determined by either measuring or computing $\boldsymbol{M}$ at at least 36 calibration positions and fitting the resulting values of a, b and c to the known calibration values of x, y and z. Equations (15) through (20) may be used subsequently to infer the Cartesian coordinates and Euler angles of moving and rotating object **10** *noniteratively* from measured values of $\boldsymbol{M}$.

Referring again to the drawings, Figure 2 is a schematic diagram of a preferred embodiment of transmission circuitry **18**. Transmission circuitry **18** is based on a

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control unit **30** that receives inputs from one or more external input ports **32** (three are shown) and from a counter **34**, and produces four different outputs: transmitted signals **TX0**, **TX1** and **TX2** directed to coils **12**, **14** and **16**, and a reset signal directed to counter **34** and a set reset flip flop (SR-FF) **36**. The identity and functionality of the other components of transmission circuitry **18** will be clear from the following description of the operation of transmission circuitry **18**.

Transmission circuitry **18** operates in two modes, reception mode and transmission mode. On startup, transmission circuitry **18** is in reception mode: a T/R line **38** from SR-FF **36** sets analog multiplexer switches **46** so that coils **12**, **14** and **16** are used as receiving antennas, the outputs of which are fed into the inputs of a trigger circuit **42**. Transmission circuitry **18** remains in reception mode until one of coils **12**, **14** or **16** receives a trigger signal. The trigger signal may be simply a short pulse which is higher than a pre-set threshold level. This is appropriate to tracking either a single object or multiple objects transmitting at separate sets of frequencies. Alternatively, if multiple objects are tracked, then each object, or each subgroup of objects, may be assigned its own modulation sequence, such as a unique digital code, to serve as a trigger signal. In this way, for example in an application to a three dimensional game, only the game pieces in play are activated. Upon receipt of such a signal, trigger circuit **42** changes the state of SR-FF **36**, changing the operational mode of transmission circuitry **18** to transmission mode. In transmission mode, SR-FF **36** sets T/R line **38** so that analog multiplexer switches **46** connect coils **12**, **14** and **16** to drivers **44**. The output of an oscillator **40** is fed via a gate **45** to counter **34**. Counter **34** is a "divide by N" counter with *N* outputs. Counter **34** counts up from zero, and the *N* outputs of counter **34** are fed into control unit **30**.

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The signals generated by control unit **30** at outputs **TX0**, **TX1** and **TX2** are periodic signals with three different fundamental frequencies, set via input ports **32**. For example, to enable the second algorithm for forming matrix $\boldsymbol{M}$, a signal with a fundamental frequency of 1000 Hz may be supplied via output **TX0**, a signal with a fundamental frequency of 2000 Hz may be supplied via output **TX1**, and a signal with a fundamental frequency of 4000 Hz may be supplied via output **TX2**. (To enable the alternative version of the second algorithm, in which, for example, the signals supplied via outputs **TX1** and **TX2** both have the same fundamental frequency, but with a difference in relative phase, the relative phase also is set via input ports **32**.) The signals may be pure sinusoids, square waves, or periodic signals of any other convenient waveform. Solving equation (2) for the matrix $\boldsymbol{c}$ is equivalent to performing a Fourier analysis of the received signals, to recover the fundamental sinusoids. After these signals have been generated for a sufficiently long time to allow reception circuitry **26** to compute the position and orientation of target **10**, control unit **30** sends a Reset signal to counter **34** and SR-FF **36** to put transmission circuitry **18** back into reception mode.

Most preferably, control unit **30** includes switches that can be operated by a user to change outputs **TX0**, **TX1** and **TX2** dynamically. For example, the signal supplied to output **TX1** or **TX2** can be changed, from a signal whose frequency is one multiple of the frequency of the signal supplied to output **TX0**, to a signal whose frequency is another multiple of the frequency of the signal supplied to output **TX0**.

Figure 3 is a schematic diagram of a simpler preferred embodiment of transmission circuitry **18**, suitable for use with the second algorithm for forming the matrix M. This embodiment operates only in the transmission mode. The output of

an oscillator **140** is fed directly to a counter **134**. Like counter **34**, counter **134** is a "divide by N" counter with *N* outputs. Counter **134** counts up from zero, and the N outputs are fed into a control unit **130** which produces three transmitted signals TX**0**, TX**1** and TX**2** that are directed to coils **12**, **14** and **16** via drivers **144**.

Figure 4 is a schematic diagram of a preferred embodiment of reception circuitry **26**. Antennas **20**, **22** and **24** are alternately and successively connected to a control/processing unit **50** via an analog selector switch **52**, an amplifier/filter **54** and an A/D converter **56**. The timing of control/processing unit **50** and A/D converter **56** is controlled by an oscillator **62**. Note that oscillators **40** and **62** need not be synchronized. Most preferably, control/processing unit **50** includes switches similar to the switches of control unit **30** described above. Also connected to, and controlled by, control/processing unit **50** is a driver **58** and an excitation antenna **60**.

To start an acquisition cycle, control/processing unit **50** sends an excitation trigger signal to excitation antenna **60** via driver **58**. Note that this form of explicit synchronization is considerably simpler than the synchronization of Raab, which requires the mixing of the received signals with a reference signal at the receiver. The trigger signal transmitted by excitation antenna **60** is received by transmission circuitry **18** of Figure 2 and causes transmission circuitry **18** of Figure 2 to flip from reception mode to transmission mode. Using analog selector switch **52**, control/processing unit **50** selects one antenna **20**, **22** or **24** at a time, thereby sampling the signals of all three antennas, at a rate sufficiently high to meet the Nyquist sampling criterion. Control/processing unit **50** computes the position and orientation of target **10**, using one of the two algorithms described above.

Figure 5 is a schematic diagram of a simpler preferred embodiment of reception circuitry **26**, for use with the embodiment of transmission circuitry **18** illustrated in Figure 3. This embodiment lacks driver **58** and excitation antenna **60**. Correspondingly, control/processing unit **150** which controls this embodiment lacks an "excite" output port. Otherwise, the embodiment of Figure 5 is identical in construction and operation to the embodiment of Figure 4.

In a most preferred embodiment of the present invention, all three spatially extended antennas **20**, **22** and **24** are coextensive, *i.e.*, they occupy substantially the same volume of space, without losing their linear independence, as taught in PCT Publication No. WO 9603188, entitled "Computerized Game Board", which is incorporated by reference for all purposes as if fully set forth herein. Particular reference is made to Figures 13A through 14E of that publication, and the accompanying description. Figure 13A shows two flat rectangular antennas **500** and **550** that, when overlapping, respond differently to transmissions from a game piece (or equivalently from target **10**), as illustrated in Figures 14A and 14B, despite the occupancy by the two antennas **500** and **550** of substantially the same volume of space. Figure 6 shows three antennas **20'**, **22'** and **24**, in the style of antennas **500** and **550** of WO 9603188, that, when superposed spatially, constitute a set of three coextensive linear independent antennas suitable for use with the present invention.

While the invention has been described with respect to a limited number of embodiments, it will be appreciated that many variations, modifications and other applications of the invention may be made.